

x runs over all the solutions to $\sum_{i=1}^n x_i^2 = N$, equidistribute on S^{n-1} for $n > 4$ as N (odd) tends to infinity. The rate of equidistribution poses however a more challenging problem. Due to its Diophantine nature the points inherit a repulsion property, which opposes equidistribution on small sets. Sarnak conjectures that this Diophantine repulsion is the only obstruction to the rate of equidistribution. Using the smooth delta-symbol circle method, developed by Heath-Brown, Sardari was able to show that the conjecture is true for $n > 5$ and recovering Sarnak's progress towards the conjecture for $n = 4$. Building on Sardari's work, Browning, Kumaraswamy, and myself were able to reduce the conjecture to correlation sums of Kloosterman sums of the following type:

$$\sum_{q \in \mathcal{Q}} \frac{1}{q} S(m; n; q) \exp(4 \pi i \sum_{p \mid mn=q} p)$$

Assuming the twisted Linnik conjecture, which states that the